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EXCHANGE RATE VOLATILITY IN THE V4 COUNTRIES

Summary

This paper deals with the analysis of the V4 countries' currencies daily exchange rates against the euro (EUR) and the US dollar (USD) for the period 1 or 4 January 1999 – 25 October 2007. The asymmetric volatility of the individual logarithmic exchange rate returns was captured using the conditional heteroscedasticity models EGARCH (1,1) – M and/or EGARCH (1,1). Taking into account these models, the static forecasts for logarithmic exchange rate returns were calculated and the forecasted values of exchange rates for the next business day (26 October 2007) were compared with the actual values.

Key words

Exchange rates, forecasts, V4 countries, conditional heteroscedasticity, EGARCH (1,1), EGARCH (1,1) – M

ACM classification

J.4 SOCIAL AND BEHAVIORAL SCIENCES, *Economics,* G.3 PROBABILITY AND STATISTICS, *Time series analysis*

JEL classification

C5 – Econometric Modeling, C53 – Forecasting and Other Model Applications

1 INTRODUCTION

The V4 countries, i.e. Czech Republic, Hungary, Poland and Slovakia are countries with the common historical and political background. All these countries became the European Union' (EU) members on 1 May 2004 and in the future plan to enter the European Monetary Union (EMU). Since the time, the majority of the "old" EU countries has adopted the EUR, the main attention in the V4 countries has been given on exchange rates against the euro (EUR) and the US dollar (USD). It was also the main reason why this paper tries to analyse the exchange rates of the V4 countries' currencies against the EUR and the USD.

In general it can be said that the exchange rates build one group of the high frequency financial time series. The common feature of those time series is a non-stationarity, which is also one of the reasons why the analysis is mainly realized on first differences of these financial time series, which are also called return series. The return series usually show high time-varying volatility and are non-normally distributed (exhibit the fat-tailed feature and leptokurtosis). In order to capture the volatility clustering of financial time series, various models of conditional heteroscedasticity have been developed.

The basic ARCH (autoregressive conditional heteroscedasticity) model first published by Engle (1982) was later generalized by Bollerslev (1986) and became the name GARCH (generalized ARCH). The non-linear EGARCH (exponential GARCH) model published by Nelson (1991) enables to capture so-called asymmetric effects (leverage effects), i.e. the different effects of positive and negative shocks on conditional volatility. The direct relationship between return and conditional variance can be captured by GARCH – M (GARCH in mean) model developed by Engle, Lilien and Robins (1987). Nowadays there exist lot of modifications of the ARCH models.[1] This paper uses the EGARCH – M model of Nelson (1991) taking into account the relationship between return and

[1] See e.g. Arlt and Arltová (2003), Franses and Dijk (2000).

conditional variance and also the ability to capture various asymmetric effects.

The whole analysis was done in econometrical software EViews 5.1 for the period 1 or 4 January 1999 – 25 October 2007 on following exchange rates: CZK/EUR, CZK/USD (2226 observations), HUF/EUR, HUF/USD (2208 observations), PLN/EUR, PLN/USD (2230 observations) and SKK/EUR, SKK/USD (2204 observations). The data were collected from web pages of individual national banks. The main aim was to capture the asymmetric volatility of the individual logarithmic exchange rate returns using the conditional heteroscedasticity model EGARCH (1,1) - M and/or EGARCH (1,1) and to calculate the static forecasts of the individual exchange rates for the next business day (26 October 2007).

2 TESTING OF THE STATIONARITY USING THE AUGMENTED DICKEY – FULLER TEST

The analysis was done on logarithmic transformation of the above mentioned time series, which were first tested on the existence of the unit root using the ADF (Augmented Dickey – Fuller) test[2]. The results of the ADF test can be found in Table 1. In case of logarithmic time series was the test applied on time series both without trend and constant, in case of time series of first differences was the test applied on time series both with trend and constant. From the results in Table 1 it is clear that it is not possible to reject the hypothesis about the existence of the unit root in any of the analysed logarithmic time series. The first differences of the analysed time series were stationary (*** indicates in whole paper significance at the 1 % significance level)[3] and it was a reason why we converted logarithmic daily exchange rates into daily rates of return by taking the difference between natural logarithms of two consecutive daily exchange rates, i.e.

$$r_{it} = d(\ln(S_{it})) = \ln(S_{it}) - \ln(S_{i(t-1)}) \quad (1)$$

where, S_{it} is the exchange rate of the *i*-th country at time t and r_{it} is the corresponding rate of return on the exchange rate.

3 MODELLING OF THE RETURN SERIES

The further analysis was done on logarithmic exchange rate returns the summary statistics of which are in Table 2. The existence of non-normality was confirmed by the values of Jarque-Bera statistics[4] taking into account the skewness and the kurtosis of the tested distribution. The calculated skewness statistics were non-zero (positive in six cases and negative in two cases, which indicates positively or negatively skewed returns distribution). The kurtosis statistics were in all cases large and positive which means that the distributions are leptokurtic relative to the normal distribution.

The individual return series are displayed in Figure 1 suggesting the presence of the high time-varying volatility. In the next step we tried to find an appropriate Box-Jenkins ARMA (autoregressive moving average)

Table 1

Time series	ADF	Time series	ADF
$\ln(CZK/EUR)_t$	-1,6061	$\ln(PLN/EUR)_t$	-0,4572
$\ln(CZK/USD)_t$	-3,7739	$\ln(PLN/USD)_t$	-0,9167
$\ln(HUF/EUR)_t$	-0,0273	$\ln(SKK/EUR)_t$	-1,6188
$\ln(HUF/USD)_t$	-0,5579	$\ln(SKK/USD)_t$	-1,3388
$d(\ln(CZK/EUR))_t$	-47,8321***	$d(\ln(PLN/EUR))_t$	-31,3031***
$d(\ln(CZK/USD))_t$	-46,4020***	$d(\ln(PLN/USD))_t$	-47,0882***
$d(\ln(HUF/EUR))_t$	-48,6516***	$d(\ln(SKK/EUR))_t$	-43,9901***
$d(\ln(HUF/USD))_t$	-47,6148***	$d(\ln(SKK/USD))_t$	-46,2479***

2 For more information see e.g. Enders (1995).

3 EViews uses the critical values determined by MacKinnon in 1996.

4 Jarque-Bera is a test statistic for testing the normality of the return series. It is distributed as a $$ chi sup 2 $$ -distribution with two degrees of freedom (see e.g. Franses and Dijk (2000)).

Table 2

	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	Jarque-Bera
$d(\ln(CZK/EUR))_t$	-0,000118	-0,000160	0,031048	-0,024805	0,003474	0,280659	9,902935	4446,810***
$d(\ln(CZK/USD))_t$	-0,000209	-0,000178	0,037654	-0,034312	0,006746	-0,026651	4,403061	182,7672***
$d(\ln(HUF/EUR))_t$	$-7{,}76.10^{-7}$	0,000000	0,046408	-0,024447	0,004569	1,327117	16,67526	17845,23***
$d(\ln(HUF/USD))_t$	$-8{,}71.10^{-5}$	-0,000125	0,050877	-0,040411	0,007541	0,145253	5,182402	445,7472***
$d(\ln(PLN/EUR))_t$	$-5{,}21.10^{-5}$	-0,000123	0,054150	-0,055269	0,006558	0,437894	10,60002	5435,726***
$d(\ln(PLN/USD))_t$	-0,000142	-0,000340	0,048066	-0,047740	0,007123	0,165982	6,048804	873,5271***
$d(\ln(SKK/EUR))_t$	-0,000115	-0,000134	0,023219	-0,032350	0,003107	0,122692	13,36280	9862,823***
$d(\ln(SKK/USD))_t$	-0,000206	-0,000215	0,026675	-0,037235	0,007003	-0,081221	3,855175	69,55156***

model[5] for the analysed return series using the autocorrelation function (ACF) and partial autocorrelation function (PACF). The mean equation of the individual return series has also the following form:

$$r_{it} = \sum_{j=1}^{p} \varphi_j r_{i,(t-j)} + \varepsilon_t + \sum_{k=1}^{q} \theta_k \varepsilon_{t-k} \qquad (2)$$

where, $\varphi_j \; (j = 1,2,\dots,p)$ and $\theta_k \; (k = 1,2,\dots,q)$ are the parameters of the appropriate ARMA(p,q) model and ε_t is a disturbance term.

From the analysed return series only the time series $d(\ln(HUF/EUR))_t$, $d(\ln(PLN/EUR))_t$ and $d(\ln(SKK/EUR))_t$ were correlated, so we used following Box-Jenkins ARMA models to ensure the uncorrelatedness:

$d(\ln(HUF/EUR))_t$ ARMA((1,4),(1,4))
$d(\ln(PLN/EUR))_t$ ARMA(3,1)
$d(\ln(SKK/EUR))_t$ ARMA(0,1).

The residuals from all return series' models (2) were tested for uncorrelatedness, homoscedasticity and normality using the q-lag Ljung-Box statistics Q(q) and $Q^2(q)$ for the levels of return and squared return series respectively[6] and Jarque-Bera statistics – the results of these tests are in Table 3.

According to the results presented in Table 3 we can say that the residuals are in all cases on the significance level 1 % till the lag 200 uncorrelated[7] and have non-normal distribution. The squared residuals are in all cases from the lag q (q = 1 or 4 or 5) correlated, which together with the high kurtosis and non-normality indicates the existence of the autoregressive conditional heteroscedasticity in the returns.

3.1 MODELLING OF THE CONDITIONAL HETEROSCEDASTICITY

To capture the existence of conditional heteroscedasticity the EGARCH (1,1) - M and/or EGARCH (1,1) were used. The EGARCH (1,1) model is as follows:

$$\ln(h_t) = \alpha_0 + \alpha_1 \frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}} + \beta_1 \ln(h_{t-1}) + \gamma_1 \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \qquad (3)$$

Table 3

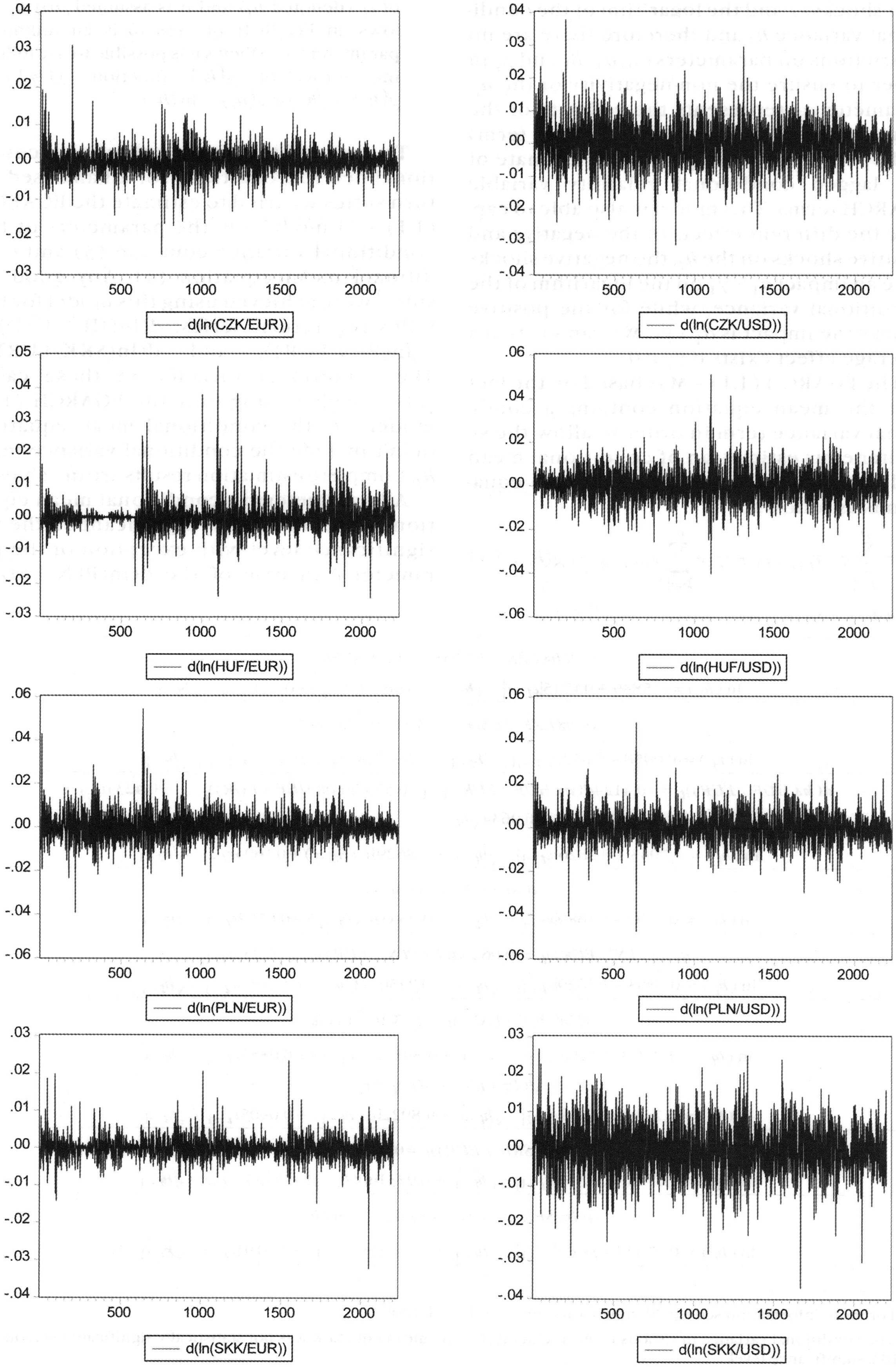
	Q(200)	Q^2 for squared residuals	Jarque-Bera
$d(\ln(CZK/EUR))_t$	183,36	$Q^2(1)$ = 88,135***	4450,523***
$d(\ln(CZK/USD))_t$	155,17	$Q^2(4)$ = 14,865***	182,538***
$d(\ln(HUF/EUR))_t$	217,42	$Q^2(1)$ = 73,009***	15802,74***
$d(\ln(HUF/USD))_t$	216,7	$Q^2(1)$ = 143,96***	445,659***
$d(\ln(PLN/EUR))_t$	217,81	$Q^2(1)$ = 228,78***	5387,186***
$d(\ln(PLN/USD))_t$	229,02*	$Q^2(1)$ = 160,47***	873,28***
$d(\ln(SKK/EUR))_t$	198,62	$Q^2(1)$ = 51,355***	9889,174***
$d(\ln(SKK/USD))_t$	178,53	$Q^2(5)$ = 20,116***	69,495***

5 See e.g. Arlt and Arltová (2003), Enders (1995), Franses and Dijk (2000).

6 Q(q) and $Q^2(q)$ follow the χ^2- distribution with q degrees of freedom.

7 In EViews it is possible to test the uncorrelatedness only till the lag 200.

Figure 1


d(ln(CZK/EUR))
d(ln(CZK/USD))
d(ln(HUF/EUR))
d(ln(HUF/USD))
d(ln(PLN/EUR))
d(ln(PLN/USD))
d(ln(SKK/EUR))
d(ln(SKK/USD))

This model describes the relation between past shocks ε_t and the logarithm of the conditional variance h_t and therefore there are no restrictions on parameters $\alpha_0, \alpha_1, \beta_1$ and γ_1 in order to ensure the non-negativity of the h_t. Parameter α_1 represents the estimate of the lagged squared residual term (ARCH term) and β_1 represents the parameter estimate of the lagged conditional volatility variable (GARCH term). This model is also able to capture the different effects of the negative and positive shocks on the h_t; the negative shocks have an impact $\alpha_1 - \gamma_1$ on the logarithm of the conditional variance, while for the positive shocks the impact is $\alpha_1 + \gamma_1$. We can say that a leverage effect exists if $\gamma_1 \neq 0$.

The EGARCH (1,1) - M is based on the fact that the mean equation contains a conditional variance term in order to allow the simultaneous estimation of conditional mean and variance.[8] The conditional mean equation is as follows:

$$r_{it} = \sum_{j=1}^{p} \varphi_j r_{i,(t-j)} + \varepsilon_t + \sum_{k=1}^{q} \theta_k \varepsilon_{t-k} + \delta g(h_t) \quad (4)$$

where $g(h_t)$ is a function of the conditional variance of ε_t (denoted h_t) and it is assumed that h_t follows an EGARCH process (δ is an unknown parameter). In EViews it is possible to use following forms of $g(h_t)$ function: $g(h_t) = h_t$, $g(h_t) = \sqrt{h_t}$ or $g(h_t) = \ln(h_t)$.

To capture the existence of the conditional heteroscedasticity for all analysed return series we tried to estimate the EGARCH (1,1) - M model, i.e. the parameters of the conditional variance equation (3) and conditional mean equation (4). Any good results[9] were achieved using this model for the following return series: $d(\ln(HUF/USD))_t$, $d(\ln(PLN/EUR))_t$ and $d(\ln(SKK/EUR))_t$. The conditional variance in these cases was modeled using only the EGARCH (1,1) model, i.e. the conditional mean equation didn't include the conditional variance term h_t. Complete estimation results are in Table 4.

All parameters in conditional mean equations were statistically significant on the 5% significance level with exception of the parameter δ in case of the $d(\ln(PLN/USD))_t$

Table 4

$d(\ln(CZK/EUR))_t = -11{,}97275 h_t$ $\ln(h_t) = -0{,}5839 + 0{,}1715\lvert\varepsilon_{t-1}\rvert/\sqrt{h_{t-1}} + 0{,}96\ln(h_{t-1}) - 0{,}041\varepsilon_{t-1}/\sqrt{h_{t-1}}$
$d(\ln(CZK/USD))_t = 3{,}41.10^{-5}\ln(h_t)$ $\ln(h_t) = -0{,}0958 + 0{,}0522\lvert\varepsilon_{t-1}\rvert/\sqrt{h_{t-1}} + 0{,}9945\ln(h_{t-1}) - 0{,}014\varepsilon_{t-1}/\sqrt{h_{t-1}}$
$d(\ln(HUF/EUR))_t = -0{,}4124 d(\ln(HUF/EUR))_{t-1} - 0{,}5537 d(\ln(HUF/EUR))_{t-4} + 0{,}4231\varepsilon_{t-1} +$ $+0{,}5260\varepsilon_{t-4} + 0{,}0634\sqrt{h_t}$ $\ln(h_t) = -1{,}7043 + 0{,}2826\lvert\varepsilon_{t-1}\rvert/\sqrt{h_{t-1}} + 0{,}8629\ln(h_{t-1}) + 0{,}1694\varepsilon_{t-1}/\sqrt{h_{t-1}}$
$d(\ln(HUF/USD))_t = \varepsilon_t$ $\ln(h_t) = -0{,}7782 + 0{,}1689\lvert\varepsilon_{t-1}\rvert/\sqrt{h_{t-1}} + 0{,}9341\ln(h_{t-1}) + 0{,}0502\varepsilon_{t-1}/\sqrt{h_{t-1}}$
$d(\ln(PLN/EUR))_t = -0{,}0623 d(\ln(PLN/EUR))_{t-3} - 0{,}055\varepsilon_{t-1}$ $\ln(h_t) = -0{,}6883 + 0{,}1629\lvert\varepsilon_{t-1}\rvert/\sqrt{h_{t-1}} + 0{,}9450\ln(h_{t-1}) + 0{,}1056\varepsilon_{t-1}/\sqrt{h_{t-1}}$
$d(\ln(PLN/USD))_t = 2{,}43.10^{-5}\ln(h_t)$ $\ln(h_t) = -1{,}215 + 0{,}2211\lvert\varepsilon_{t-1}\rvert/\sqrt{h_{t-1}} + 0{,}8955\ln(h_{t-1}) + 0{,}0856\varepsilon_{t-1}/\sqrt{h_{t-1}}$ $d(\ln(PLN/USD))_t = \varepsilon_t$ $\ln(h_t) = -1{,}243 + 0{,}2196\lvert\varepsilon_{t-1}\rvert/\sqrt{h_{t-1}} + 0{,}8922\ln(h_{t-1}) + 0{,}0905\varepsilon_{t-1}/\sqrt{h_{t-1}}$
$d(\ln(SKK/EUR))_t = 0{,}0987\varepsilon_{t-1}$ $\ln(h_t) = -1{,}0708 + 0{,}2535\lvert\varepsilon_{t-1}\rvert/\sqrt{h_{t-1}} + 0{,}9231\ln(h_{t-1}) + 0{,}0353\varepsilon_{t-1}/\sqrt{h_{t-1}}$
$d(\ln(SKK/USD))_t = 3{,}0410^{-5}\ln(h_t)$ $\ln(h_t) = -0{,}3762 + 0{,}0665\lvert\varepsilon_{t-1}\rvert/\sqrt{h_{t-1}} + 0{,}9674\ln(h_{t-1}) + 0{,}0191\varepsilon_{t-1}/\sqrt{h_{t-1}}$

8 For more information see e.g. Blenman, Chatterjee and Ayadi (2005).

9 The conditional variance in various forms included into the mean equation was not statistically significant even on the 10% significance level.

which was significant only on the 6,9% significance level. As a result of the low significance of the δ parameter in this model, also the EGARCH (1,1) model was used to capture the conditional heteroscedasticity.

The parameters in individual conditional variance equations were all significant on the 1% significance level, the only exception was the parameter γ_1 in the conditional variance model for the return series $d(\ln(SKK/USD))_t$ which was significant on the 3,61% significance level. From the above mentioned results it is clear that the existence of conditional heteroscedasticity taking into account the asymmetric effects was confirmed in all cases.

In the next step we tested the standardized residuals from all the analysed models (using the Ljung-Box statistics Q(q) and $Q^2(q)$ and Jarque-Bera statistics), and these were considered to be already uncorrelated, homoscedastic, but non-normally distributed. This means that the models were well chosen, but in consequence of the non-normality could the estimation results be described as to be consistent only as quasi-maximum likelihood estimations.

4 CALCULATION OF STATIC FORECASTS

All models from Table 4 were used to calculate the static forecasts of the analysed return series for period *t* considering the information available in period *t*-1. The calculated static forecasts of the individual logarithmic exchange rate returns together with the conditional standard deviation are depicted in Figure 2[10].

According to the Figure 2 we can say that the individual logarithmic exchange rate returns were oscillating around the zero. Furthermore it is clear that if we omitted the existence of the conditional heteroscedasticity, the corresponding parameter estimates would be ineffective and the confidence interval wouldn't take into account the time-varying variance.

The static forecasts of the analysed exchange rates for the next business day (26 October 2007) together with the actual values are in Table 5. The calculated exchange rate values were in all cases greater than the actual values, but the differences were very small – the maximal difference was 1,17%.

Table 5

Exchange rate	Forecasts	Actual values	Diff. in percentage
CZK/EUR	27,1122	26,96	-0,56%
CZK/USD	18,9522	18,743	-1,12%
HUF/EUR	251,0458	251	-0,02%
HUF/USD	175,79	174,69	-0,63%
PLN/EUR	3,6459	3,6247	-0,58%
PLN/USD[11]	2,5515 2,5522	2,5227	-1,14% -1,17%
SKK/EUR	33,4052	33,257	-0,45%
SKK/USD	23,4699	23,247	-0,96%

5 CONCLUSION

Forecasting of the exchange rates' future values using various exchange rate models has always been a very popular and interesting issue in order to be able to make a clever financial decision. The possible volatility of the exchange rate plays an important role by forecasting. In connection to this fact it is necessary to mention that the conditional volatility models enables to foresee the behaviour of the conditional volatility and therefore the conditional mean models together with the conditional volatility models are good instruments for forecasting of financial time series.

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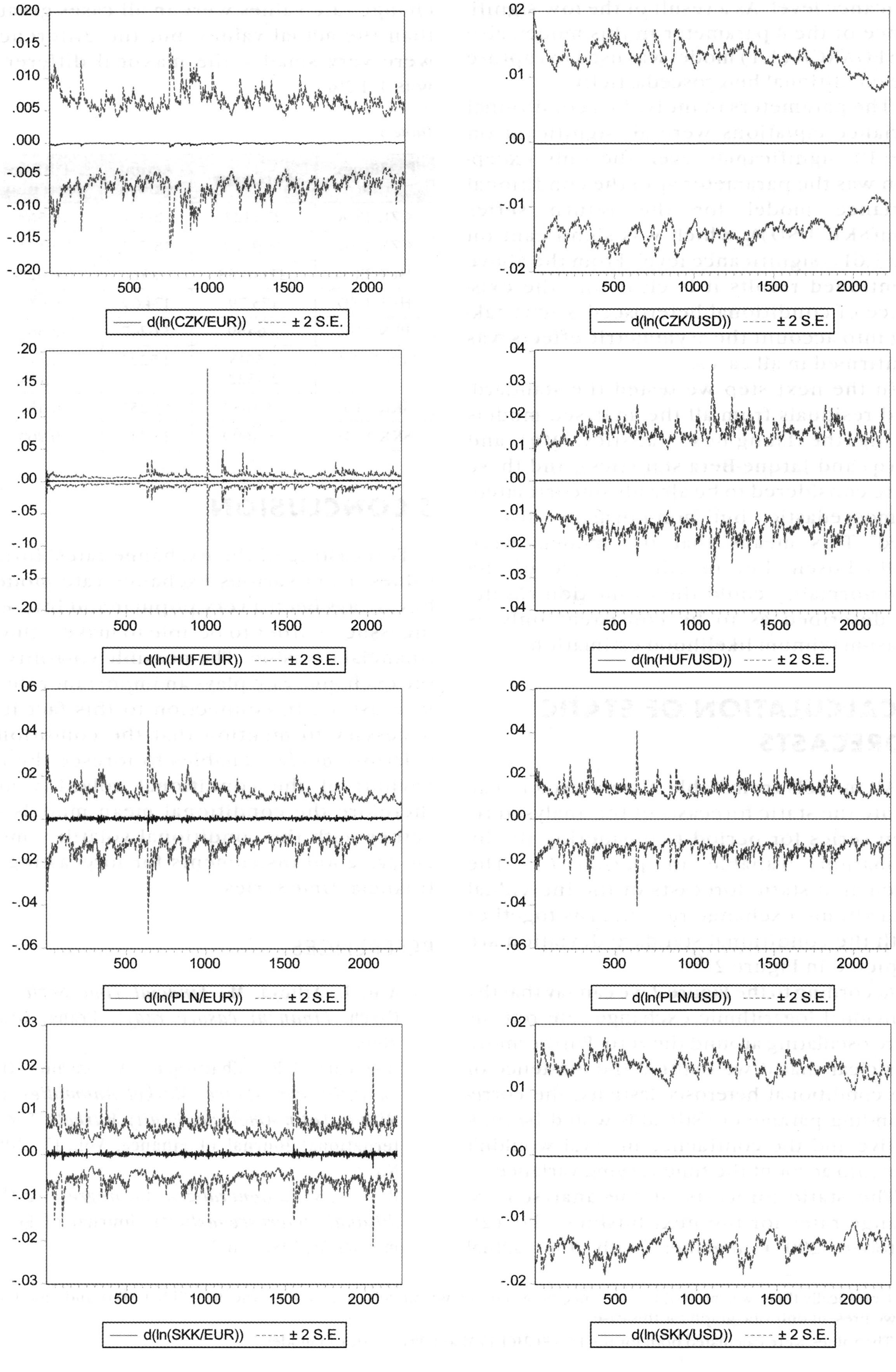
[10] Graphically there was no visible difference between the two models estimated for the $d(\ln(PLN/USD))_t$ and therefore we present only one graph for this case.

[11] The values were calculated both for the EGARCH(1,1)-M and EGARCH(1,1) model.

Figure 2


d(ln(CZK/EUR)) ± 2 S.E.
d(ln(CZK/USD)) ± 2 S.E.
d(ln(HUF/EUR)) ± 2 S.E.
d(ln(HUF/USD)) ± 2 S.E.
d(ln(PLN/EUR)) ± 2 S.E.
d(ln(PLN/USD)) ± 2 S.E.
d(ln(SKK/EUR)) ± 2 S.E.
d(ln(SKK/USD)) ± 2 S.E.

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Biography:

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